ABSTRACT

Decisions made during conceptual design can have a major impact on the success of a design project. However, the inherently imprecise nature of design is a major source of uncertainty and risk in conceptual design decisions. A single concept relates to a large set of specific design implementations, each of which has a different level of desirability based on the tradeoffs designers are willing to make. It therefore is beneficial for designers to have an understanding of the various tradeoffs they can achieve by implementing a concept.

In this paper, we describe an approach to modeling design concepts under uncertainty based on a tradeoff space representation. We use the principles of decision making to develop a useful interpretation of a tradeoff space for decisions under uncertainty and to identify criteria useful for eliminating undesirable tradeoffs from consideration. We illustrate our approach to modeling and decision making on an example for the conceptual design of a gearbox.

INTRODUCTION

Deciding between competing design concepts is a critical step in a system development process and can, to a large extent, dictate the overall success of a project. Intuitively, design activities later in a process cannot compensate for the selection of a poor concept. Some researchers note that such early-process decisions are important because it becomes progressively more expensive to rectify errors later [1-3]. Others argue, primarily based on studies of Toyota Motor Company and its suppliers, that improvements in early-phase decision making can lead to improved product quality and reduced design cycle times [4-6]. However, despite a clear motivation for improved conceptual design decisions, how to do this in a way that is both practically efficient and fundamentally sound remains an open question.

The main challenge is that a design concept is not a completely-defined solution to a problem; it is a general approach to implementing a function or system. Although the concept “diesel engine” implies certain aspects about the design solution, it leaves unresolved numerous implementation details. From this perspective, a design concept is an abstraction of a large set of possible design implementations [7]. Decision methods that rely on detailed engineering analysis, which requires a precise definition of a design, are poorly suited for the set-based nature of design concepts.

Consider again the diesel engine concept: How much does it cost? What is its reliability? How much power can it output? The answer to these questions is that it depends: different implementations will have different attributes, reflecting different tradeoffs a designer can make. Even if an engineering analysis model could be perfectly accurate, the imprecise, set-based nature of a design concept would lead to uncertainty in the analysis results and expose a decision maker to considerable risk.

In this paper, we investigate an alternative to approaches that are based on analyzing imprecise design definitions. Under our approach, one considers the set-based nature of a design concept explicitly. Since any number of a large set of implementations are possible for any given concept, we model the tradeoffs that designers can achieve should they implement a particular concept. Thus, we describe a concept in terms of the possible consequences of selecting it rather than what it is in physical terms.

This investigation is part of a broader vision for conceptual design, in which designers compose system-level concepts from reusable tradeoff-space models of common functional components. For example, system engineers could model different configurations for a hybrid gas-electric power train quickly and confidently if they begin from concept tradeoff models of the main
functional units (power storage, power conversion, etc.). This would enable them to incorporate detailed tradeoff considerations into their decision making without requiring specialized domain knowledge about every component concept. Our focus in the current paper is on how to model and decide between concepts in isolation, leaving the composition of such models for future work.

In prior work, we have demonstrated an analogous approach for modeling design concepts under deterministic assumptions [8]. Although the general strategy is similar to what we use here, the approach we describe in the current paper is a significant departure due to the added complexity of decision making under uncertainty. In both works, we model a design concept in terms of the tradeoffs designers can achieve by implementing it. However, the deterministic notion of a tradeoff space is incompatible with decisions under uncertainty. In this paper, we contribute a new, general interpretation of what constitutes a tradeoff space and apply it in the context of concept modeling. Our interpretation is based on accepted decision theory and is meaningful independent of our modeling approach.

Another similarity is that in both works we generate a model for a design concept using data about those implementations of the concept that a designer might choose rationally, called the parameterized efficient set. However, the presence of uncertainty complicates the process of identifying this set. To identify a parameterized efficient set, one must eliminate design implementations that are technically feasible but inferior from a tradeoff perspective. The elimination criterion we use in the prior work, called Pareto dominance, is inappropriate for decisions under uncertainty. In this paper, we draw upon the decision theory literature for tests—called stochastic dominance criteria—appropriate for decisions under uncertainty. Although stochastic dominance can be difficult to evaluate in general, we develop a specialized criterion that can be useful in many engineering situations and apply it in our modeling approach.

We demonstrate our approach on the conceptual design of a gearbox. We consider three concepts for implementing the gearbox, each with different physical configurations. For each concept, we identify a parameterized efficient set and fit a predictive tradeoff model to it. Using these models in concert with decision preferences, we select the most preferred concept. We verify that our models provide accurate tradeoff predictions and yields the correct decision by comparing the results to results from a baseline method based on traditional engineering optimization techniques.

The remainder of this paper is organized as follows. First, we survey the problem background, including prior work on concept modeling and selection. Next, we describe the proposed approach for modeling design concepts. This is followed by a description of how to formulate decisions using the tradeoff-space models. We then demonstrate our approach and verify it relative to a baseline method. Finally, we discuss the contributions, limitations and potential future work.

### PROBLEM BACKGROUND

#### DECIDING BETWEEN DESIGN CONCEPTS

In the context of mechanical design, conceptual design involves the transformation of requirements and objectives into a definition for the basic solution structure that designers will refine in subsequent phases of a design process [9, 10]. In the broader context of systems design, conceptual design corresponds to the definition of an abstract structure for a system in terms of its functional subunits, their configuration into a system, and requirements for the design of each subunit [2, 11-14]. In either case, conceptual design entails many tasks, including functional decomposition, concept generation, and decision making, and the outcome of a conceptual design decision is an imprecise definition of the final system.

In this paper, we consider how designers can make decisions between two or more concepts. We focus on concepts for which several prior implementations exist, such that designers have access to information about these implementations or understand the principles underlying them. This includes common functional components that can be implemented using different technologies or physical configurations. Pumps, motors, speed reducers and actuators are examples of these. We anticipate that the types of models described in this paper can form a basis for composing concepts for novel systems by composing non-novel component-level concepts, and we plan to study this in future work.

A basic premise for this paper is that designers should make decisions systematically and using methods that are sound with respect to the accepted norms of decision theory. This perspective is called Decision-Based Design (DBD) in the literature [15-18]. In particular, we build upon the decision theoretic framework of Multi-Attribute Utility (MAUT) [19], which is an extension of the utility theory by von Neumann and Morgenstern [20] to the case of multiple competing objectives. Using MAUT, designers can express their preferences with regard to tradeoffs under uncertainty and, when combined with their beliefs about those uncertainties, can identify the most preferred decision alternative.

A generic decision is formulated in MAUT as

\[
  a^* = \arg \max_{a \in \mathcal{A}} E[\mu(x_a)],
\]

(1)
where \( \mathcal{A} \) is the set of feasible decision alternatives, \( a \in \mathcal{A} \) is a specific alternative, \( x_a \in \mathbb{R}^n \) is a random vector of attributes for alternative \( a \) having distribution function \( F_{x_a} : \mathbb{R} \to \mathbb{R} \) is a suitably defined utility function, and \( E[\cdot] \) is the expectation operator. In this context, an attribute, is a measure of progress toward a decision objective \([21, 22]\). For example, in a choice between a diesel engine and a gasoline engine, attributes may be cost, reliability and measures of technical performance (e.g., power output, max torque).

Engineers typically formulate conceptual design decisions using engineering analysis models, which map a description of a system to the relevant decision attributes. In general, this mapping is different for each concept because different concepts can involve different operating principles. One can express conceptual design decision formulated using analysis models as

\[
c^* = \arg \max_{c \in C} E[u(f(c))] ,
\]

where \( c = C \) is a design concept, \( \phi_c \) is vector of variables describing the concept as required for analysis and \( f(\cdot) \) represents the engineering analysis models that relate the description to a distribution in the space of design attributes.

A significant challenge in making conceptual design decisions is that concepts typically are defined too imprecisely to support accurate analysis. In terms of the decision formulation given above, in a parametric description, \( \phi_c \), might include only a fraction of the design variables required to describe a finalized design implementation. Thus, a unique \( \phi_c \) vector might correspond to a large set of design implementations, each of which could, if designers could consider them individually, have a different expected utility. Even if designers had a perfect analysis models, they would have significant uncertainty in their evaluation of a concept.

Designers always can reach a decision using the generic formulation of Equation (2), but the imprecise nature of design concepts exposes them to considerable risk. What is more, most solution methods include no mechanism for dealing with the inherently uncertain and abstract nature of design concepts. Thus, designers who use these methods may neglect this source of uncertainty and, consequently, expose themselves to even greater risk.

PRIOR RESEARCH

A few methods from the DBD community are targeted for use on conceptual design decisions, but these have their own limitations. Chen and coauthors describe a method in which a designer models uncertainty about the final implementation of a concept explicitly using uniform probability distributions applied to the design space variables and makes decisions using a modified robust design formulation \([23, 24]\). Although the method is easy to apply, it constrains how designers can formalize their beliefs and preferences. Furthermore, they provide no procedure for determining the bounds of the required uniform distributions.

Wood and coauthors describe a method for abstracting the characteristics of partially-defined solutions from a database of prior design instances \([25, 26]\). They perform abstraction by constructing a "design PDF" from the data and "integrating out" unneeded variables. However, their approach rests on the use of this "design PDF," for which they provide no concrete interpretation or justification.

Mattson and Messac demonstrate an approach to selecting design concepts based on a Pareto set representation \([27, 28]\). They represent a concept using the Pareto set of possible implementations, which has some similarities with the approach we present here. However, theirs is not a well-formed decision method—rather than formalizing decision preferences they rely on dominance reasoning, which can fail to identify a single concept as the most preferred (a situation called indeterminacy). Furthermore, their approach for dealing with uncertainty and risk lacks foundation in accepted decision theory.

The notion of Set-Based Design (SBD) commonly is associated with conceptual design. Under a SBD approach, designers focus on eliminating inferior implementations from a set of designs, and possibly delay a decision in favor of gathering more information \([5]\). Most SBD methods eliminate designs using interval-or set-based constraint propagation methods (e.g., the methods of \([29-31]\) and the tools demonstrated in \([32, 33]\)). Although there is evidence of analogous practices in industry \([4]\) and that these practices are beneficial \([6]\), SBD does not constitute a comprehensive decision method—by focusing on elimination rather than selection, SBD can lead to indeterminacy. Some research exists on extending the set-based design perspective to include eliminations based on preference information \([7, 34]\). Although this improves the odds of identifying a most preferred design, there are no assurances designers will avoid indeterminacy.

Several approaches to deciding between design concepts are practiced in the systems design and engineering community, the more common of which include quality function deployment \([35, 36]\), Pugh selection \([37, 38]\), and the analytic hierarchy process \([39, 40]\). Hazelrigg analyzes these and other strategies, finding them each to be lacking in light of accepted results from decision theory \([16]\). Another systems-oriented decision approach, known as analytical target cascading, is effective at propagating design requirements down to subsystems, but assumes the system configuration is known \([41-43]\).
MODELING DESIGN CONCEPTS

Our strategy for modeling a design concept is to abstract information about implementations of the concept in a way that is favorable to decision making. Concepts are modeled:

• as relationships in a suitably defined tradeoff space;
• using predictive models that capture association rather than causation;
• using information about only the implementations that a designer might rationally choose; and
• in a way that is reusable across multiple decision problems.

Using the fitted models, designers can predict the most preferred tradeoff achievable by implementing a particular concept. They make conceptual design decisions according to a two-step process of:

1. identifying the most preferred tradeoff achievable by each concept, and
2. selecting the concept that can lead to the most preferred tradeoff overall.

In prior work, we justified this general strategy and demonstrated an approach for modeling design concepts assuming designers can evaluate tradeoffs with certainty [8]. Uncertainty introduces considerable complications, and extending the strategy to decisions formulated in MAUT is nontrivial. In this section, we explain the main characteristics of our general strategy and the extensions necessary for MAUT.

TRADEOFF SPACES

Designers can realize a particular design concept using any number of physically distinct implementations. This complicates attempts to formalize a space of design variables, and motivates us to find a modeling approach that avoids design space representations altogether.

We use the term tradeoff space to mean a representation of the possible tradeoffs a designer might make. A tradeoff space relates to designer objectives, but is independent of the alternatives being considered. Every design implementation maps to exactly one point in the tradeoff space, and designers can compare two designs by applying their preferences to the tradeoff points for each design. Figure 1 is an illustration of the difference between using analysis models and tradeoff space models.

The tradeoff space relates closely to the concept of a decision making attribute. In the case of decisions with deterministic tradeoffs, the tradeoff space is the space of attributes. For decisions in MAUT the tradeoff space is the space of probability distributions over the attribute space. This follows from the axiomatic definition MAUT as decisions between lotteries (see [22] for a development of this theory) and is evidenced by the procedures for eliciting preferences in MAUT, which involves comparisons of lotteries over attributes (as opposed to comparisons of precise values).

Strictly speaking, the notion of a “space” of probability distributions is ill-defined. However, it is possible to visualize a restricted space of distributions, assuming, for example, that all random variables are normally distributed. Under such assumptions it is possible to define a variable space that describes the permissible distributions. For instance, one can define a normal distribution unambiguously in terms of its mean and variance; several other distributions can be described in a similar fashion. In this paper, we assume all distributions are normal.

Our definition for a tradeoff space makes intuitive sense when one considers the kinds of tradeoffs that occur when uncertainty is involved. For example, when product quality is a concern it is common for designers to trade mean performance to achieve reduced variability in that same attribute. One can visualize this tradeoff in the mean-variance space of that attribute. Figure 2 is an illustration of the relationship between the
attribute space and the tradeoff space for MAUT decision problems.

PREDICTIVE MODELING

There is growing recognition that, fundamentally, engineering design activities are part of an overall business enterprise, and that designers must make decisions in this context [44-47]. Some authors even advocate a decision approach based on explicit modeling and simulation of enterprise activities [48, 49]. However, we are skeptical about whether the gains of this would outweigh the costs associated with such a modeling effort. Instead, we adopt a strategy based on predictive modeling.

A predictive model is something with which users can draw inferences about the unknown value of one variable given the values of other variables [50, 51]. The power of predictive models is that they permit useful inference in cases where the underlying causal mechanism is unknown or too complex to formulate analytically. Although currently more common in the context of marketing and finance, such models also are useful for engineering design, where designers can use predictive modeling techniques to infer relationships too complex to construct directly from engineering principles. A common example of this is statistical cost estimation [52-58].

Our strategy is to model a concept by fitting a predictive model to tradeoff space data about prior implementations of the concept. The fitting can be achieved using standard regression analysis or interpolation methods. The motivation for using information about prior implementations is that this information will include enterprise considerations implicitly. Although this is a departure from the traditional approach of engineering analysis, the idea of generating tradeoff-space predictive models has been demonstrated previously in the contexts of power transmission [59], environmentally benign design [60], and motorized scooters [61].

To express the model and its role mathematically, let \( \theta \) denote a particular tradeoff in the \( m \)-dimensional space of tradeoffs, \( \Theta \). A predictive model, \( M(\cdot) \), is formulated to use \( p < m \) dimensions of the tradeoff space to infer the remaining \( m - p \) dimensions. Thus, one can express a complete vector in the tradeoff space as

\[
\theta = [\theta', M(\theta')],
\]

where \( \theta' \) denotes the \( p \)-dimensional vector of tradeoff dimensions that are used as independent variables in the predictive model. The selection of which dimensions to use as independent variables is arbitrary, and one should structure the model so it yields the best predictions possible.

STOCHASTIC DOMINANCE AND EFFICIENCY

Designer Rationality, Efficiency and Dominance

To model all feasible tradeoffs that correspond to a design concept would be unproductive, since a rational designer never would choose most of these implementations. Therefore, it is beneficial to distinguish between feasible and "good" tradeoffs. Two decision theoretic concepts—efficiency and dominance—are helpful for making this distinction.

Our strategy is to model only the tradeoffs that are efficient, which means that designers cannot improve in any tradeoff criteria without sacrificing another. It is common to identify the efficient tradeoffs by performing dominance tests. An alternative, \( A \), is said to be dominate another, \( B \), when decision makers can guarantee that \( A \) is preferred to \( B \). A dominated design cannot be efficient, so one can identify the efficient set using dominance tests.

For deterministic decision problems, the set of efficient alternatives is known commonly as the Pareto set [22, 62]. Pareto sets have been used in a number of frameworks for decision making in design [63-67], including concept selection [8, 27]. However, the notion of Pareto dominance is tied inherently to deterministic decision making. Some authors have attempted to extend Pareto dominance to decisions under uncertainty (e.g., [28]), but the appropriate approach, in light of the norms of decision theory, requires an expanded interpretation of dominance.

Stochastic Dominance

To identify the efficient set for decisions formulated in MAUT, designers must appeal to the general notion of stochastic dominance [68, 69] (sometimes called probabilistic dominance [22], though some have used...
that term differently to mean that one design has a high probability of dominating another, e.g. [70, 71]). Research involving stochastic dominance criteria is limited mostly to the economics and operations research literature; we are unaware of prior applications in the context of engineering design.

A stochastic dominance test involves comparing distribution functions that are defined over an attribute space—i.e., they are tradeoff-space evaluations. A stochastic dominance test also requires assumptions about the mathematical structure of the corresponding utility function. This is similar to the monotonicity assumption associated with the deterministic concept of Pareto dominance, but the useful assumptions are more varied in the stochastic case owing to the greater complexity of preferences for decisions under uncertainty. Researchers have identified a number of stochastic dominance conditions for different classes of utility functions and assumptions about the uncertainties involved [72-76].

Table 1: Summary of common classes of utility functions, their relationships and the relevant stochastic dominance criteria.

<table>
<thead>
<tr>
<th>Class</th>
<th>Defining Assumptions</th>
<th>Interpretation</th>
<th>Associated Dominance Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_0$</td>
<td>none</td>
<td>All utility functions.</td>
<td>n/a</td>
</tr>
<tr>
<td>$U_1$</td>
<td>$U_1 := {u \in U_0 \mid \frac{du(x)}{dx} \geq 0}$</td>
<td>Monotonically non-decreasing.</td>
<td>First-degree stochastic dominance (FSD).</td>
</tr>
<tr>
<td>$U_2$</td>
<td>$U_2 := {u \in U_1 \mid \frac{d^2u(x)}{dx^2} \leq 0}$</td>
<td>Monotonically non-decreasing and non-risk-taking (i.e., risk neutral or risk averse).</td>
<td>Second-degree stochastic dominance (SSD).</td>
</tr>
<tr>
<td>$U_3$</td>
<td>$U_3 := {u \in U_2 \mid \frac{d^2u(x)}{dx^2} \geq 0}$</td>
<td>Monotonically non-decreasing, non-risk-taking and decreasing absolute risk aversion.</td>
<td>Third-degree stochastic dominance (TSD).</td>
</tr>
</tbody>
</table>

In this paper, we demonstrate concept modeling using a multi-attribute extension of the $U_2$ preference class—i.e., utility functions that are monotonically non-decreasing and risk averse. As with our prior work for deterministic decisions, target-type non-monotonic objectives are permitted via a parameterization of the efficient set. Intuitively, the risk averseness assumption seems reasonable for engineering design problems. However, in practice one should validate such an assumption more concretely.

Researchers have generalized the stochastic domination criteria listed in Table 1 to the case of multi-attribute decisions, but the conditions are not always straightforward extensions of the single-attribute tests [77, 78]. In general, stochastic dominance tests for multi-attribute problems involve comparisons of multivariate distribution functions. Evaluating stochastic dominance criteria can be challenging in practice [79], but some special cases exist in which it is computationally straightforward. In this paper, we take advantage of two such simplifying assumptions: independence and normality.

When attribute distributions are statistically independent, one can test any multi-attribute dominance condition by testing the corresponding single-attribute condition using the marginal distributions [75]. Although this assumption may not hold perfectly for many engineering problems, it is a reasonable approximation in a number of cases. Thus, we adopt it in this paper and leave the more general case to future work.

In cases where an attribute is normally distributed, the second-degree stochastic dominance (SSD) test simplifies to a comparison of the means and variances of the distributions [72, 73, 80], which significantly reduces computational complexity. As with statistical independence, this assumption is not valid in general, but is an effective approximation in many practical cases and we adopt it in this paper.

To formulate the multivariate form of SSD as we use it in this paper, we first must define some notation. Let $\mathbf{x}$ be an $n$-dimensional attribute vector for a decision.
problem. Also, let \( a \) and \( \bar{a} \) be the decision alternatives with distribution functions \( F_a(\cdot) \) and \( F_{\bar{a}}(\cdot) \), respectively. Finally, let \( \mu_i \) and \( \sigma^2_i \) denote the mean and variance, respectively, of the \( i^{th} \) attribute of alternative \( a \) and \( \mu_i \) and \( \sigma^2_i \) denote the mean and variance, respectively, of the \( i^{th} \) attribute of alternative \( \bar{a} \). Thus, the mean-variance dominance rule we use is stated:

**Multivariate SSD (MV Case):** If \( F_a(x) \) and \( F_{\bar{a}}(x) \) are independent and normally distributed, then \( F_a(x) \) dominates \( F_{\bar{a}}(x) \) by multivariate SSD if and only if \( \mu_i \geq \bar{\mu}_i \) for all \( i = 1 \ldots n \) and \( \sigma^2_i \leq \bar{\sigma}^2_i \) for all \( i = 1 \ldots n \) and at least one of the inequalities is strict.

Figure 3 is an illustration in mean-variance space of this dominance rule as applied in the single-attribute case. Under the assumptions we use, Multivariate SSD always results in a curve or surface in mean-variance space.

**REUSABILITY THROUGH PARAMETERIZATION**

An underlying assumption of SSD is that preference is monotonic in each attribute. This type of objective is common in design, with examples including the minimization of manufacturing cost or maximization of operational lifetime. However, many problems involve preferences that are non-monotonic in certain attributes. Important categories of non-monotonic objectives include what one can describe as “target seeking” or “goal seeking” objectives, and their complement, “target avoiding” objectives. For such objectives, a decision maker’s preferences are maximized (or minimized) at a particular value of the associated attribute and diminish (increase) at points away from this target. Target-seeking objectives often arise in the context of interfaces or interactions between systems and when system-level attributes are translated into the attributes of lower-level subsystems; target-avoiding objectives can arise when trying to prevent undesirable resonance and noise effects.

Non-monotonic objectives are problematic from a reuse perspective because it is impossible to test for domination without problem-specific information. In contrast, monotonic objectives often hold across a wide range of problems—e.g., designers nearly always prefer less cost. Our strategy is to identify parameterized efficient sets—efficient sets identified as a function of the attributes associated with target-seeking or target-avoiding objectives. Predictive models are fit to the parameterized efficient set information for a particular concept. At the time designers seek to make a particular decision, they know the appropriate target values and the predictive model reduces to a model of the appropriate efficient set.

Two considerations for applying the parameterization strategy are that:

1. one must sample the parameter(s) at several levels and identify an efficient set for each sample, and
2. when fitting a predictive model to the parameterized efficient set data, one must use the parameter(s) as independent variable(s).

We described this strategy originally in the context of deterministic decision problems as a novel extension to the classical Pareto set, and demonstrated the reusability of the resulting models by solving two different decision problems with the same predictive model [8]. In the present paper, we apply it to efficient sets identified using the stochastic dominance criteria.

**FORMULATING CONCEPTUAL DESIGN DECISIONS USING TRADEOFF MODELS**

We assume that for each concept designers consider, they have a predictive tradeoff model that has been fitted to parameterized efficient set information and that any assumptions underlying the models are consistent with their preferences and beliefs. Our formulation of a conceptual design decision in terms of predictive tradeoff models is based on the generic MAUT decision problem expressed in Equation (1). However, the general mathematical formulation involves some specialized notation.

The first step of the decision process is for designers to estimate the most preferred tradeoff designers can achieve using each concept. This is followed by designers selecting the concept that leads to the most preferred tradeoff overall.

**ESTIMATING TRADEOFFS ACHIEVABLE BY A DESIGN CONCEPT**

Let \( x_\theta \) denote the random vector defined over the attribute space corresponding to the tradeoff vector \( \theta \). The elements of \( \theta \) contain the parameters of the distribution for the random vector. For example, for a single-attribute problem with a normally distributed attribute, one could define \( \theta = [\mu, \sigma^2] \) and this would
imply that $x_0 \sim N(\mu, \sigma^2)$. One therefore can express the expected utility of tradeoff $\theta$ as $\mathbb{E}[u(x_0)]$.

To estimate the most preferred tradeoff achievable by a design concept, one can search the corresponding predictive model to find the tradeoff that maximizes expected utility. One can express this as

$$Eu_c^* = \max_{\theta \in \Theta_c} \mathbb{E}[u(x_{\theta, M_c(\theta)})], \quad (3)$$

where $M_c(\cdot)$ denotes the predictive model associated with design concept $c$, $\Theta_c$ denotes the feasible set for tradeoff dimensions used as independent variables in the predictive model, and $Eu_c^*$ is the maximum expected utility value.

Equation (3) constitutes an optimization under uncertainty problem and designers can solve it using various techniques. Although analytical simplifications or solution procedures may sometimes be viable, the predictive model typically will have a simple algebraic structure that, in most cases, makes numerical solution procedures attractive. In our experience, it is effective to couple standard nonlinear programming techniques with sampling-based uncertainty quantification methods (i.e., Monte Carlo simulation) and variance reduction techniques (e.g., Latin hypercube sampling [81], common random numbers to improve gradient estimates [82]).

SELECTING THE MOST PREFERRED DESIGN CONCEPT

To select the most preferred concept, designers compare the maximum expected utility values for each concept. Mathematically, one can state this in terms of the results from Equation (3) as

$$c^* = \arg \max_{c \in C} Eu_c^* \quad (4)$$

where $c \in C$ is a particular design concept. The number of distinct concepts designers consider typically is small, so this search is fast.

DEMONSTRATION: SELECTING A GEARBOX CONCEPT UNDER UNCERTAINTY

OVERVIEW AND OBJECTIVES

We demonstrate our approach to making conceptual design decisions under uncertainty in the context of a gearbox design problem. Our objectives are to illustrate the approach and verify that it produces reasonable results. To achieve the latter objective, we compare our approach to a baseline method that is assured of identifying the concept most preferred under MAUT but that is computationally intensive and not generally viable in practice.

Our example consists of three concepts for implementing a gearbox, each with a different physical configuration and design space representation. Although the design scenario is simple, it is adequate to demonstrate and verify our approach. What is more, the simplicity is what makes it feasible to compare our approach to the extensive search method.

DESIGN SCENARIO PRELIMINARIES

System and Environment

Our gearbox design problem is situated in the broader context of a small, single-person off-road vehicle. The components relevant to this problem are its engine, continuously-variable transmission (CVT), a fixed-ratio gearbox and a rear differential with a fixed gear ratio, arranged as depicted in Figure 4. The task is to select a design concept for the fixed-ratio gearbox assuming the other system components already have been determined. Designer preferences are defined at the system level and propagated downward to the gearbox subsystem in order to select a design concept.

Table 2 is a summary of various system and environmental parameters that affect vehicle performance. All uncertainties are assumed independent and modeled using normal distributions.
Concepts and Their Domain in the Design Space

We assume the preexisting vehicle components constrain the gearbox to have co-axial input and output shafts that rotate in the same direction. We consider three concepts for the gearbox design:

- **Planetary Gearbox (PGB):** Basic planetary gear system, with input on sun, output on arm and fixed ring (Figure 5(a)).
- **Single-Sided Fully-Reverted Gearbox (SGB):** Four-gear system with two identical pinions and two identical gears (Figure 5(b)).
- **Double-Sided Fully-Reverted Gearbox (DGB):** Similar to single-sided concept, but includes two paths for torque flow (Figure 5(c)).

Each concept is an abstraction of many possible implementations that conform to a particular structure. Within each concept, designs have a common design space representation, but this representation differs among the concepts. The design variables that distinguish implementations of the same concept control the number of teeth on each gear, the gear face widths and the gear module. Other design parameters, such as gear material, quality factor, etc., are assumed the same for all concepts; it is possible to vary these, but it would add little to the demonstration.

All three concepts are defined over a wide domain in their respective design spaces. The number of teeth on any gear is allowed to vary from 15 to about 50. The face width, constant for all gears in the same gearbox, is permitted to vary from 6.35 mm to 8.75 cm. Gear module can take on any of the 25 standard Series 1 values, which range from 0.1 mm to 5 cm.1

**GENERATION OF PREDICTIVE TRADEOFF MODELS**

The tradeoff model we generate is based on automatically synthesized design implementations. We analyze each implementation using standard engineering analysis and uncertainty propagation methods. We then identify the parameterized efficient set by using the SSD criterion to eliminate dominated implementations. Finally, we fit a predictive model to the parameterized efficient set data.

**Tradeoff Space Definition**

Our tradeoff space is based on three decision attributes:

- **Gear ratio:** Transformation ratio from input to output. Problem-dependent, target-seeking preferences.
- **Reliability:** Probability that the gearbox operates without failure, considering static and dynamic loading phenomena. Prefer more reliability to less, all other factors being equal.
- **Cost:** Construction costs of gearbox, computed as a function of the material and parts involved. Prefer less cost to more, all other factors being equal.

Of these attributes, reliability and cost involve significant uncertainties, which we assume are normally distributed and independent. However, one can compute gear ratio

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1 Source: [http://www.qtcgears.com/Q410/QTC/Q410P337.htm](http://www.qtcgears.com/Q410/QTC/Q410P337.htm)
with certainty. Thus, the resulting tradeoff space consists of five dimensions:

- Gear ratio, $N_g$
- Mean reliability, $\mu_R$
- Variance of reliability, $\sigma_R^2$
- Mean cost, $\mu_C$
- Variance of cost, $\sigma_C^2$

Gear ratio and reliability can be computed from the design variables using standard engineering analysis models (see e.g., [83]). Cost is an empirical relationship fit from catalog data. The distribution parameters for reliability and cost are estimated by propagating the uncertain parameters listed in Table 2 through the analysis models using Latin hypercube sampling [81].

**Design Instances and Stochastic Domination**

Because we lack a suitable source of information prior implementations of these concepts and their operating principles are well understood, we can generate implementations automatically via systematic sampling within the respective design spaces of each concept. Each implementation is checked for technological feasibility—e.g., vetted against geometric constraints—and the feasible ones are analyzed using standard engineering models to predict the attributes of interest.

Preferences associated with cost and reliability are monotonic, meaning the related tradeoff space dimensions are subject to stochastic dominance filtering. Preferences associated with gear ratio are non-monotonic and, in this example, depend on system-level attributes such as vehicle top speed and acceleration. Thus, gear ratio serves as a parameter when identifying the parameterized efficient set. For each concept, the gear ratio parameter is sampled across its domain and we apply the SSD criterion to eliminate dominated implementations at each parameter level.

**Formalization of the Decision Problem**

**System-level Decision Preferences**

In this example, we consider a scenario involving a race for which the winner is awarded prize money. This dictates designer preference at the system level, which are to maximize expected profit (i.e., net value). The corresponding utility function is

$$u(R, W, C) = RW - C,$$

where $R$ is the reliability of the gearbox, $W$ is the anticipated winnings assuming perfect reliability and $C$ is the cost of building the gearbox. Anticipated winnings is computed using a formula determined using data from similar races and is a function of vehicle performance attributes, including maximum speed and acceleration. The overall decision objective is to maximize the expected value of Equation (5).

Because all aspects of the vehicle design are fixed except the gearbox, winnings is essentially a function of the gear ratio.

**Decision Formulation using Predictive Tradeoff Models**

We formulate the procedure for estimating the most preferred tradeoff achievable by each concept based on Equation (3). However, we have the complication that decision preferences are formulated in terms of system level tradeoffs and the models for our concepts are defined in terms of component-level tradeoffs. Thus, we must define a transformation between the two tradeoff spaces.

We can denote this transformation generally as

$$x_{sys} = f_{sys}(x_{comp}, \xi),$$

where $x_{comp}$ is a random vector defined over the attribute space of the component, $\xi$ is a random vector of external uncertainties (such as the items listed in Table 2), and $x_{sys}$ is the corresponding random vector defined over the system-level attributes.

This transformation is straightforward in the current example. We have equated system-level cost and reliability to the corresponding component-level attributes. The only model required is to compute winnings as a function of gear ratio, and the only external uncertainty involved is the uncertainty due to this model.

Our tradeoff models are formulated to compute the mean cost, $\mu_C$, as a function of the other tradeoff criteria. Thus, in keeping with the notation of Equation (3), let $\theta' = [N_g, \mu_R, \sigma_R^2, \sigma_C^2]$ denote the independent
predictor vector and \( \mu_c = M_c(\theta') \) denote the predictive model for a concept, \( c \in \{\text{SGB, DGB, PGB}\} \). Given these, the procedure for estimating the expected utility of most preferred tradeoff within a particular concept is formalized as:

\[
Eu_c^* = \max_{\theta' \in \Theta} \mathbb{E} \left[ u \left( f_{\text{sys}} \left( x_{[\theta', M_c(\theta')]} , \xi_W \right) \right) \right],
\]

where \( x_{[\theta', M_c(\theta')]} \) is the vector of component-level attributes—the deterministic gear ratio and the random variables reliability and cost—and \( \xi_W \) is a random variable representing uncertainty in the winnings model. We solve this expression for each concept using standard nonlinear programming and uncertainty propagation techniques.

A final decision is made by selecting the concept with the largest \( Eu_c^* \) value.

Baseline Decision Formulation

The baseline procedure involves searching the concept design spaces directly to identify the most preferred implementation of each concept. This is a direct implementation of engineering optimization and uncertainty propagation methods and, in principle, yields good estimates of the most preferred design implementation for each concept. To ensure the veracity of our estimates, we conduct several random-restart trials of each optimization (at least 10 of each) and keep the best results. This helps ensure we avoid local minima. The results of the optimization runs allow us to determine whether the approach based on parameterized efficient sets (a) leads to selecting the most preferred concept and (b) whether the estimate of the most preferred tradeoff achievable by this concept is accurate.

For a particular concept, \( c \in \{\text{SGB, DGB, PGB}\} \), the search for the expected utility of the most preferred implementation is formulated mathematically as:

\[
Eu_c^* = \max_{\phi_c \in \Phi_c} \mathbb{E} \left[ u \left( f_c \left( \phi_c, \xi \right) \right) \right]
\]

where \( \phi_c \) is vector of design variables specifying the implementation, \( \Phi_c \) is the set of feasible design implementations, \( \xi \) is a random vector of uncertain parameters corresponding to Table 2, and \( f_c(\cdot, \cdot) \) represents the engineering analysis under uncertainty that calculates the gear ratio and distribution parameters for winnings and cost corresponding to implementation \( \phi_c \) and uncertainty \( \xi \). Note that \( f_c(\cdot, \cdot) \) represents the same analysis used to evaluate the design instances used for fitting the parameterized efficient set models.

A before, the most preferred concept is the one for which \( Eu_c^* \) is maximal.

DECISION RESULTS

Table 3 contains results from the gearbox concept selection problem. The table contains the tradeoff criteria and expected utilities corresponding to the most preferred instance of each design concept as predicted using the predictive tradeoff models. The final decision is in favor of the PGB concept because estimates indicate that it leads to the largest expected utility. To continue development of this concept, designers can use the tradeoff criteria indicated in the table as design-to-targets.

The extensive search yields the results listed in Table 4, and confirms the results based on tradeoff space search. The decision approach formulated using predictive tradeoff models yields the same decision, to develop the PGB concept, as one would reach using the more computationally intensive extensive search. Furthermore, estimates for the most preferred tradeoff achievable by each concept are essentially the same as what we find using the extensive search approach (differences of only a few percent). This indicates that the tradeoff models yield the right decision for the right reason.

<table>
<thead>
<tr>
<th>Expected Utility, ( Eu_c^* )</th>
<th>Tradeoff Space Search Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGB</td>
<td>1185 1078 1134</td>
</tr>
<tr>
<td>DGB</td>
<td>7.3 7.3 7.5</td>
</tr>
<tr>
<td>SGB</td>
<td>5.76 5.76 5.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gear Ratio, ( N_g )</th>
<th>( \text{PGB, DGB, SGB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.99</td>
<td>5.733 5.76 5.76</td>
</tr>
<tr>
<td>1.99</td>
<td>5.76 5.76 5.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reliability mean, ( \mu_R )</th>
<th>Extensive Search Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGB</td>
<td>0.99 0.985 0.989</td>
</tr>
<tr>
<td>DGB</td>
<td>0.013 0.02 0.014</td>
</tr>
<tr>
<td>SGB</td>
<td>0.013 0.02 0.014</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost mean, ( \mu_C )</th>
<th>Extensive Search Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGB</td>
<td>275 372 320</td>
</tr>
<tr>
<td>DGB</td>
<td>7.5 7.5 7.5</td>
</tr>
<tr>
<td>SGB</td>
<td>7.5 7.5 7.5</td>
</tr>
</tbody>
</table>

Table 3: Results from parameterized efficient set model for gearbox design under uncertainty. The most preferred concept is the PGB.

Table 4: Results from extensive search for gearbox design under uncertainty. The most preferred alternative is in the PGB concept.
DISCUSSION

The approach to modeling a design concept using a predictive tradeoff-space representation of its parameterized efficient set holds promise, but the results of this paper are insufficient to provide conclusive support for its practical viability. The modeling approach is based on the principles of MAUT, which ensures that designers can make decisions in a sound fashion. Furthermore, results from the gearbox example problem indicate that, in principle, designers can use the models to predict accurately the best tradeoff they can achieve by implementing a particular concept. However, in practice, designers will encounter complications that may limit the effectiveness of the approach. These situations require further study.

One such complication is that uncertainties present in parameterized efficient set information will, in general, differ from what designers encounter when making decisions with the resulting model. In the gearbox example, we use the uncertainty models in Table 2 when analyzing implementations prior to model fitting and in solving the baseline search method, which is intended to provide a "true" result against which we can compare the new approach. By using the same uncertainty models in both places, we provide strong evidence for the internal consistency of our approach and its effectiveness under relatively ideal circumstances. However, further study is required to assess the effectiveness of the approach in general.

Another potential complication is that, in general, attributes are not distributed normally and independently. We used these relatively restrictive assumptions in order to formalize the tradeoff space and construct a computationally efficient stochastic dominance criteria. As it turns out, the assumptions did not hold in the gearbox example—although the underlying uncertainties were normal and independent, the attributes had a slight correlation and a normal distribution was not a particularly good fit to the reliability attribute data—but the results were positive none the less. This indicates that further study is needed to determine precisely how sensitive tradeoff estimates and decision results are to these assumptions. Research on more general tradeoff space representations and stochastic dominance criteria also would be beneficial.

A key advantage of the proposed modeling approach is that it avoids the use of imprecise design-space definitions of a concept. By using a tradeoff-space representation, all concepts for satisfying a particular function have a common representation. In the example, we are able to represent the three physically distinct gearbox concepts in the same tradeoff space. We anticipate this being particularly useful in the context of composing system-level concept models from lower-level models because designers will be able to abstract all concepts for a particular functional unit into a single tradeoff model. For example, we could do this with the gearbox concepts by combining the data about their parameterized efficient sets and then find a new parameterized efficient set that represents them all. The main benefit of this is that designers can minimize the combinatorial explosion that occurs when considering system-level configuration problems. We plan to demonstrate this in future work.

Although we did not demonstrate it in this paper, the gearbox concept models are reusable across a number of decision problems. This is possible due to the idea of parameterizing the efficient set in terms of the target-type objectives, which we originally demonstrated elsewhere. Without the parameterization, designers would require decision-specific information to identify the efficient set.

Also critical to our modeling approach are the two new innovations that we introduce in this paper—the general interpretation of what constitutes a tradeoff space in the context of decisions under uncertainty and the use of stochastic dominance to eliminate irrelevant implementations of a concept. Both are based on established principles for making decisions under uncertainty and, we believe, useful to engineering designers beyond our particular modeling approach.

SUMMARY

In this paper, we discussed how designers can model a concept under uncertainty by formalizing tradeoff-space relationships in the parameterized efficient set of concept implementations. We illustrated a particular approach for modeling concepts in this way and for making conceptual design decisions using these models. In the process of developing this approach, we contributed several new ideas, including a general interpretation of a tradeoff space for decisions in MAUT and the application of stochastic dominance in the context of design decisions. By comparing the results of decisions made under our approach with those from a baseline decision method, we observe that our approach yields accurate estimates of the tradeoffs achievable by each concept and leads to a correct decision. However, our illustration is too limited to constitute conclusive validation and our approach includes several assumptions that may limit its practical usefulness. Thus, we conclude that further study is warranted. Avenues for future research includes demonstrating the approach on more complex problems, relaxing some assumptions, and demonstrating the compositional modeling of system-level design concepts.

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